

# Critique of materials-based models of ductile machining in brittle solids

Han Huang<sup>1</sup>  | Brian R. Lawn<sup>2</sup> | Robert F. Cook<sup>2</sup>  | David B. Marshall<sup>3</sup>

<sup>1</sup>School of Mechanical and Mining Engineering, The University of Queensland, Brisbane, QLD, Australia

<sup>2</sup>Material Measurement Laboratory, National Institute of Standards and Technology, Gaithersburg, MD, USA

<sup>3</sup>Materials Science and Engineering Program, University of Colorado, Boulder, CO, USA

## Correspondence

Han Huang, School of Mechanical and Mining Engineering, The University of Queensland, QLD 4072, Australia.  
Email: han.huang@uq.edu.au

## Funding information

Australia Research Council, Grant/Award Number: DP180103275

## Abstract

Indentation fracture models of ductile machining in hard and brittle materials are critically appraised. Relations obtained in a seminal study for critical depths of cut below which fracture is suppressed are examined and amended. Limitations inherent in any such materials-based analysis, in addition to uncertainties in empirical measurements of underpinning mechanical properties (modulus, hardness, toughness) and of threshold grinding depths, suggest that caution should be exercised in unconditional usage. Notwithstanding these limitations, the value of the indentation fracture methodology in placing ductile machining on a sound materials science footing is maintained.

## KEYWORDS

critical depth of cut, ductile machining, hard and brittle materials, indentation fracture

## 1 | INTRODUCTION

A seminal ductile grinding model for hard and brittle solids with ionic-covalent bonding (ceramics, glasses, semiconductors) proposed by Bifano, Dow and Scattergood (BDS)<sup>1</sup> has become a centerpiece in the machining research community. This highly cited model draws from indentation fracture mechanics to predict the critical depth of cut below which material can be removed without residual microcracks extending into the sublayer. It offers guidance on optimizing conditions for shaping smooth surfaces in a cost- and time-economical manner. The value of that work is that it places machining on a solid materials-based foundation, in terms of independently measured material properties: elastic modulus  $E$ , hardness  $H$ , and toughness  $K_c$  or equivalent fracture energy  $R$ . The manufacturing community consistently uses the BDS relation that emerges from that model as a tool for quantifying the ductile machining process.

## 2 | APPRAISAL

The starting point for BDS is an equation from a 1976 paper defining the threshold indentation size  $a$  (Figure 1) below

which attendant cracking about a fixed-profile contact deformation zone is suppressed:<sup>2</sup>

$$a_c \sim ER/H^2 \quad (1)$$

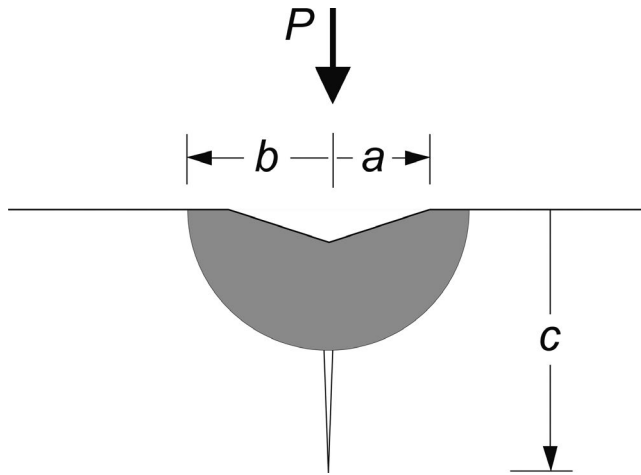
This equation reflects a dimensional difference between competing plastic and fracture processes, the first volume-controlled and the second area-controlled. BDS then posit a relationship between fracture energy and toughness

$$R \sim K_c^2/H \quad (2)$$

The appearance of  $H$  in this last equation comes from the allusion that cracks may exhibit a plastic zone around their tip. Then assuming proportionality between critical depth of cut  $d_c$  and threshold indentation size  $a_c$ , BDS obtain

$$d_c = \lambda (K_c/H)^2 \quad (3)$$

with  $\lambda = 0.15(E/H)$ . The constant 0.15 is evaluated by BDS from a correlation plot of predictions from Equation 3 against empirical measurements of crack-free depths in diamond-ground subsurfaces. In indentation theory, the ratio  $H/K_c$



**FIGURE 1** Contact damage in hard and brittle solid with sharp, fixed-angle indenter, according to expanding cavity model. Indenter at normal load  $P$  leaves impression half-diagonal  $a$  surrounded by hemispherical plastic zone of radius  $b$  and produces cracks of length  $c$

has the characteristic of an 'index of brittleness',<sup>2</sup> and  $E/H$  an 'index of plasticity'.<sup>3</sup>

Some questions may be raised concerning the BDS analysis. The first comes from the appearance of hardness  $H$  in the denominator in Equation 2. In fact, cracks in hard and brittle solids do not have plastic tip zones: they are atomically sharp.<sup>4</sup> In that case, the fracture mechanics relation between fracture energy and toughness has a familiar form<sup>5,6</sup>

$$R \sim K_c^2/E \quad (4)$$

Thus modulus  $E$  replaces  $H$ , which means that the quantity  $E/H$  would disappear from Equations 1 and 3.

A second question concerns the variability of parameters  $E$ ,  $H$ , and  $K_c$ , as well as the accuracy of experimentally measured critical grinding depths, used by BDS to calibrate coefficient  $\lambda$  in that equation. Their calibration is based on data for a limited group of brittle materials, cited in Bifano's 1988 thesis,<sup>7</sup> with the  $E/H$  term confined to a narrow range 13 to 17. This is a small subset of  $E/H$  values for a broader range of brittle materials (Table 1), so the calibration does not critically test the dependence on this term. Moreover, the literature reveals a considerable spread in quoted values of the parameters for a given material, especially toughness  $K_c$ . Such material quantities vary widely with the tests used to measure them, and are also sensitively dependent on factors such as microstructure, crystallography, composition, and fabrication history, rendering specification of the constant in  $\lambda$  unreliable. BDS acknowledge that extrapolation of customarily measured  $K_c$  and  $H$  values down to sub- $\mu\text{m}$  scales is problematic, especially in materials with so-called R-curves, ie those whose toughness increases with continuing crack extension.<sup>1</sup>

### 3 | AMENDMENT

Indentation analysis evolved during the 1970s, and the work used in the BDS modeling derives from that era. A more rigorous 1980 fracture mechanics analysis of cracking associated with sharp, fixed-profile indenters introduces a different dependency on  $E/H$ .<sup>8</sup> The 1980 analysis was based on a 'pressurized expanding cavity' representation of the elastic-plastic stress field. A hemispherical plastic zone of radius  $b$  surrounding a contact of surface dimension  $a$  creates the field that drives ensuing cracks of size  $c$ , depicted in Figure 1.<sup>8</sup> The classical relations between dimensions  $a$  and  $c$  and indenter load  $P$  for materials with well-developed median/radial cracks have the form

$$P/a^2 \sim H \quad (5)$$

$$P/c^{3/2} \sim (H/E)^{1/2} K_c \quad (6)$$

The validity of Equations 5 and 6 has been confirmed in a recent review by Cook.<sup>9</sup> A threshold condition for crack suppression may be obtained by setting  $a_c = c_c$  and eliminating load  $P$  from these equations:

$$a_c \sim (H/E) (K_c/H)^2 \quad (7)$$

which differs from the BDS formalism in its dependence on the  $H/E$  term.

There is a further twist. According to the expanding cavity model the depth of the indentation plastic zone is determined by dimension  $b$ , not  $a$ . Appendix B in the 1980 paper<sup>8</sup> gives an expression for these relative dimensions, which includes another term in  $E/H$ :

$$b \sim a (E/H)^{1/2} \quad (8)$$

for any given fixed-profile indenter. Postulating that the depth of cut  $d$  should scale with  $b$ , combination of Equations 7 and 8 yields

$$d_c = \Lambda (K_c/H)^2 \quad (9)$$

in analogy to Equation 3, with  $\Lambda = 8.7(H/E)^{1/2}$  another dimensionless coefficient. The constant 8.7 is determined by equating Equations 3 and 9 at  $E/H = 15$ , representing a mid-point value within the  $E/H$  range 13 to 17 used in BDS to calibrate Equation 3.<sup>7</sup> So the factor  $E/H$  reappears in the  $d_c$  coefficient, but in inverse square root form.

### 4 | ANALYSIS

The basic forms of Equations 3 and 9 are the same in relation to the brittleness term  $K_c/H$ , differing only in the  $E/H$

TABLE 1 Representative properties of select hard and brittle materials\*

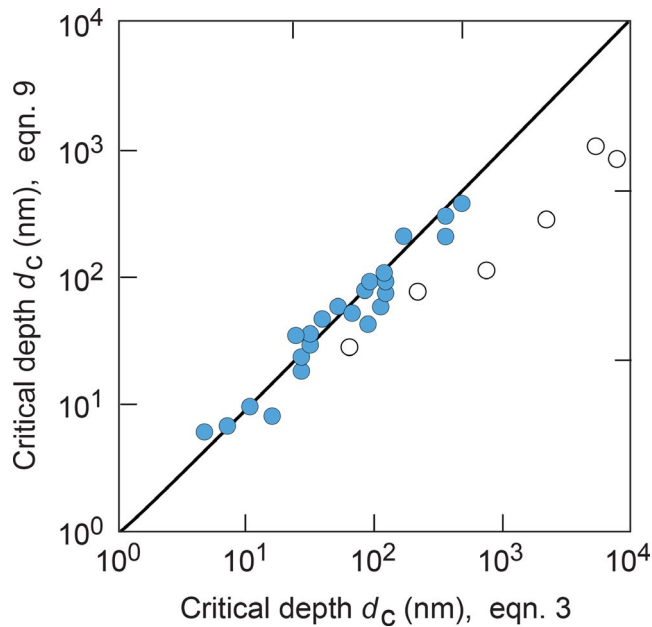
Material	Modulus $E$ (GPa)	Hardness $H$ (GPa)	Toughness $K_c$ (MPa.m <sup>1/2</sup> )	Plasticity Index $E/H$	Brittleness Index $H/K_c$ (μm <sup>-1/2</sup> )	$d_c$ (Equation 9) (nm)	$d_c$ (Equation 3) (nm)
Glass (fused silica)	79	6.8	0.8	12	8.6	35	24
Diamond (sc)	1000	80	4.0	13	20	6	5
Glass (soda-lime)	70	5.5	0.7	13	7.9	40	31
Glass-ceramic (pyroceram)	109	8.4	2.5	13	3.4	214	172
Boron nitride (pc)	450	33	5.2	14	6.3	58	51
Glass (lead alkali)	65	4.9	0.7	14	7.2	46	38
Glass (arsenic trisulphide)	17	1.1	1.7	15	4.2	90	91
Boron carbide (pc)	500	32	6.0	16	5.3	77	82
Glass-ceramic (lithium disilicate)	95	5.8	2.2	16	2.6	309	353
Germanium (sc)	140	9.0	0.5	16	18	7	7
Silicon (sc)	168	11	0.7	16	15	10	10
Mullite (pc)	225	14	1.6	16	8.8	28	31
Silicon nitride (pc, hot-pressed)	300	19	4.0	16	4.6	101	114
Silicon carbide (pc)	400	24	2.5	17	9.6	23	27
Silicon nitride (pc, react. bond.)	170	10	2.0	17	4.8	90	115
Zirconia (3Y-TZP) (pc)	210	12	5.2	18	2.3	391	493
Sapphire (sc)	425	22	2.1	20	10	18	27
Alumina (pc, fine grain)	400	20	4.0	20	5.0	78	120
Glass-ceramic (C-MGC)	63	3.0	1.0	21	3.0	211	350
Gallium nitride (sc)	284	12	0.8	24	15	8	16
Magnesium fluoride (pc)	140	5.8	0.9	24	6.4	43	87
Lanthanum phosphate (pc)	133	5.6	1.0	24	5.6	57	114
Magnesium oxide (pc)	240	9.3	1.2	26	7.8	29	64
Alumina (pc 10% glass)	390	13	2.9	30	4.5	78	219
Tungsten carbide (pc)	575	13	12	44	1.1	1089	5400
Zinc sulphide (pc)	98	1.9	0.6	52	3.2	121	772
Zinc oxide (pc)	120	2.0	1.0	60	2.0	281	2250
Zinc selenide (pc)	68	1.0	0.9	68	1.1	855	8260

\*Data collated from earlier studies.<sup>6,8,10,13-17</sup> sc, single crystal; pc, polycrystal. [Correction added July 15, 2020, after first online publication: Plasticity Index for Silicon nitride (pc, react. bond.) was changed from 1/8 to 17]

dependence of the respective coefficients  $\lambda$  and  $\Lambda$ . The calibration of  $\Lambda$  is open to refinement, reliant as it is on the choice of an intermediate  $E/H$  from BDS, as well as on the veracity of the original calibration of  $\lambda$  from empirical detection of subsurface cracks in actual machined surfaces for a limited set of materials and grinding conditions. Moreover, while it can be argued that Equation 9 is based in more rigorous indentation theory, representations of the elastic-plastic field used to quantify the  $E/H$  dependence in that theory are subject to their own approximations and assumptions. Quasiplastic zones in hard and brittle solids are more complex than the best continuum elastic-plastic models, even in homogeneous, isotropic solids.<sup>10-12</sup> Complications arising from microstructural

diversity, compositional variations, crystallographic anisotropy, and strain hardening, as well as stress intensifications from translational contact, are missing in such models.

To compare the original BDS Equation 3 with the amended Equation 9, representative material property data for a wide selection of well-behaved hard and brittle solids from past studies are compiled in Table 1, values of  $E/H$  listed in ascending order. By well-behaved, we mean materials that display the classical surface radial crack patterns implicit in the above indentation analysis, those without significant R-curves. The values of  $E/H$  in Table 1 extend well outside the range in BDS. It is apparent that the greatest divergence between Equations 3



**FIGURE 2** Comparison plot of predicted depth of cut  $d_c$ , Equation 9 vs Equation 3. Data with filled symbols indicate materials with  $E/H < 25$ , unfilled symbols materials with  $E/H > 25$  [Color figure can be viewed at [wileyonlinelibrary.com](http://wileyonlinelibrary.com)]

and 9 will occur for those materials lower in the Table. Predictions of critical depths  $d_c$  from Equation 9 are included in Table 1, along with those from BDS Equation 3. A comparative plot of these predicted values is displayed in Figure 2. For the bulk of the materials, those with  $E/H < 25$  (filled symbols), the difference between the two predictions is less than a factor of 2. For those softer outliers beyond this range (unfilled symbols), the progressive divergence begins to cast serious doubt on the capacity of any ductile machining relation, original or amended, to provide an estimate of  $d_c$  to much better than an order of magnitude.

## 5 | CONCLUSION

Extreme caution needs to be exercised in any prediction of critical depths for ductile machining from analytical relations, BDS or amended. We suggest that the materials processing community should be aware of limitations and assumptions in such relations before unconditional usage. Actual cutting depths are sensitive to factors other than just material properties—eg grit or tool shape (sharp or blunt) and size (large or small), machining conditions such as cutting speed, lubricating medium, temperature, etc—none of which are explicitly included in Equations 3 or 9. A systematic tabulation of experimental data relating to these additional factors under different machining conditions would be useful in refining calibration of the coefficients in Equations 3 and 9.

At best, relative evaluations from competing analytical relations of this kind are probably not more accurate than a factor of 2; at worst, not more than an order of magnitude. Absolute predictions are even less reliable. Nevertheless, the virtue of the original BDS relation and its amended form in Eq. 9 in providing insight into the role of key material properties underlying ductile machining remains undiminished.

## ACKNOWLEDGEMENT

Financial support for this program was provided to HH by the Australia Research Council (ARC) through the Discovery Projects Scheme (DP180103275).

## ORCID

Han Huang  <https://orcid.org/0000-0003-3353-2970>

Robert F. Cook  <https://orcid.org/0000-0003-0422-8881>

## REFERENCES

1. Bifano T.G., Dow T.A., Scattergood R.O.. Ductile-regime grinding: a new technology for machining brittle materials. *Trans ASME J Eng Indust.* 1991;113(2):184–9.
2. Lawn B.R., Jensen T., Arora A.. Brittleness as an indentation size effect. *J Mater Sci.* 1976;11(3):573–5.
3. Lawn B.R., Howes V.R.. Elastic recovery at hardness indentations. *J Mater Sci.* 1981;16(10):2745–52.
4. Lawn B.R., Hockey B.J., Wiederhorn S.M.. Atomically sharp cracks in brittle solids: an electron microscopy study. *J Mater Sci.* 1980;15(5):1207–23.
5. Irwin G.R.. Fracture. In: Flügge S., editor. *Handbuch der Physik*, vol. 6. Springer-Verlag: Berlin; 1958. p. 557–94.
6. Lawn B.R.. *Fracture of Brittle Solids*, 2nd edn. Cambridge: Cambridge University Press; 1993.
7. Bifano T.G.. *Ductile-Regime Grinding of Brittle Materials*. PhD Thesis, North Carolina State University, 1988.
8. Lawn B.R., Evans A.G., Marshall D.B.. Elastic/plastic indentation damage in ceramics: the median/radial crack system. *J Amer Ceram Soc.* 1980;63(9–10):574–81.
9. Cook R.F.. A critical evaluation of indentation crack lengths in air. *J Amer Ceram Soc.* 2020;103(4):2278–95.
10. Chiang S.S., Marshall D.B., Evans A.G.. The response of solids to elastic/plastic indentation I: stresses and residual stresses. *J Appl Phys.* 1982;53(1):298–311.
11. Chiang S.S., Marshall D.B., Evans A.G.. The response of solids to elastic/plastic indentation II: fracture initiation. *J Appl Phys.* 1982;53(1):312–7.
12. Fischer-Cripps A.C., Lawn B.R.. Stress analysis of contact deformation in quasi-plastic ceramics. *J Amer Ceram Soc.* 1996;79(10):2609–18.
13. Lawn B.R., Marshall D.B.. Hardness, toughness, and brittleness: an indentation analysis. *J Amer Ceram Soc.* 1979;62(7–8):347–50.
14. Marshall D.B., Noma T., Evans A.G.. A simple method for determining elastic-modulus-to-hardness ratios using Knoop indentation measurements. *J Amer Ceram Soc.* 1982;65(10):C175–176.
15. Marshall D.B.. Geometrical effects in elastic/plastic indentation. *J Amer Ceram Soc.* 1984;67(1):57–60.

16. Kaliszewski M.S., Behrens G., Heuer A.H., Shaw M.C., Marshall D.B., Dransman G.W., et al. Indentation studies on  $Y_2O_3$ -stabilized  $ZrO_2$ : I development of indentation-induced cracks. *J Amer Ceram Soc.* 1994;77(5):1185–93.
17. Davis J.B., Marshall D.B., Housley R.M., Morgan PED. Machinable ceramics containing rare-earth phosphates. *J Amer Ceram Soc.* 1998;81(8):2169–75.

**How to cite this article:** Huang H, Lawn BR, Cook RF, Marshall DB. Critique of materials-based models of ductile machining in brittle solids. *J Am Ceram Soc.* 2020;103:6096–6100. <https://doi.org/10.1111/jace.17344>